HKN ECE 310 Exam 1 Review Session

Corey Snyder

Benjamin Eng

LSIC Systems

- Linearity
  - Satisfy Homogeneity and Additivity
  - Can be summarized by Superposition
    - If $x_1[n] \rightarrow y_1[n]$ and $x_2[n] \rightarrow y_2[n]$, then $ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$
- Shift Invariance
  - If $x[n] \rightarrow y[n]$, then $x[n - n_0] \rightarrow y[n - n_0]$ $\forall$ n_0 and $x[n]$
- Causality
  - Output cannot depend on future input values

BIBO Stability

- Three ways to check for BIBO Stability:
  - Pole-Zero Plot (more on this later)
  - Absolute summability **of the impulse response**
  - Given $|x[n]| < \alpha,\ if\ |y[n]| < \beta < \infty$, then the system is BIBO stable
    - A bounded input $x[n]$ yields a bounded output $y[n]$
    - Ex: $y[n] = x^5[n] + 3$ vs. $y[n] = x[n] * u[n]$
- Absolute Summability
  - $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

Impulse Response

- $y[n] = x[n] * h[n]$
  - $h[n]$ is the impulse response
- System output to an $x[n] = \delta[n]$ input
  - $h[n] = \delta[n] * h[n]$
- $Y(z) = H(z)X(z)$
  - Convolution in the time/sample domain is multiplication in the transformed domain, both the z-domain and frequency domain.

Convolution

- $y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$
- System must be:
  - Linear
  - Shift Invariant
- Popularly done graphically
- Can also be done algebraically

Z-Transform

- We mainly focus on the one-sided, or unilateral, z-transform
- $X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$
- Typically perform inverse z-transform by inspection or by Partial Fraction Decomposition
- Important properties:
  - Multiplication by n: $nx[n] \leftrightarrow -z(\frac{dX(z)}{dz})$
  - Delay Property #1: $y[n-k]u[n-k] \leftrightarrow z^{-k}Y(z)$
- Make sure to note the Region of Convergence (ROC) for your transforms!
  - More in the next slide!
- DTFT is only defined if the ROC contains the unit circle

BIBO Stability Revisited

- Pole-Zero Plot
  - For an LSI system: if the ROC contains the unit circle, this system is BIBO stable
  - The ROC is anything greater than the outermost pole if the system/signal is causal
  - The ROC is anything less than the innermost pole if the system/signal is anti-causal
  - If we sum multiple signals, the ROC is the **intersection** of each signal's ROC
  - What if the ROC is $|z| > 1$ or $|z| < 1$?
    - This is *marginally stable*, but unstable for ECE 310 purposes
  - For unstable systems, you are commonly asked to find a bounded input that yields an unbounded output. Few ways to do this:
    - Pick an input that excites the poles of the system.
    - If the system's impulse response $h[n]$ is not absolutely summable, $\mathrm{u}[n]$ will work
    - $\delta[n]$ frequently works too, like when $h[n]$ is unbounded, e.g. $h[n] = 2^n u[n]$

Discrete Time Fourier Transform

$$X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)e^{j\omega n}d\omega$$

- Important Properties:
  - Periodicity!
  - Linearity
  - Symmetries (Magnitude, angle, real part, imaginary part)
  - Time shift and modulation
  - Product of signals and convolution
  - Parseval's Relation
- Know your geometric series sums!

Frequency Response

- For any stable LSI system: $H_d(\omega) = H(z)|_{z=e^{j\omega}}$
- What is the physical interpretation of this?
  - The DTFT is simply the z-transform evaluated along the unit circle!
  - It makes sense that the system must be stable and LSI since the ROC will contain the unit circle, thus ensuring that the DTFT is well defined
- Why is the frequency response nice to use in addition to the z-transform?
  - $e^{j\omega}$ is an *eigenfunction* of LSI systems
    - $h[n] * Ae^{j\omega_0} = \lambda Ae^{j\omega_0} = AH_d(\omega_0)e^{j\omega_0}$
  - By extension: $x[n] = \cos(\omega_0 n + \theta) \rightarrow y[n] = |H_d(\omega_0)| \cos(\omega_0 n + \theta + \angle H_d(\omega_0))$

Magnitude and Phase Response

- Very similar to ECE 210
- Frequency response, and all DTFTs for that matter, are 2π periodic
- Magnitude response is fairly straightforward
  - Take the magnitude of the frequency response, remembering that $|e^{j\omega}| = 1$
- For phase response:
  - Phase is "contained" in $e^{j\omega}$ terms
  - Remember that cosine and sine introduce sign changes in the phase
  - Limit your domain from $-\pi$ to π.
- For real-valued systems:
  - Magnitude response is even-symmetric
  - Phase response is odd-symmetric

LSIC Examples

- For the following systems, determine whether it is linear, shift-invariant, and causal

- $y[n] = x^2[n]$

- $y[n] = x[|n|]$

- $y[n] = 3^{-|n|} \log(|x[n]| + 1)$

Impulse Response and Convolution Examples

- Given $x[n] = [6\,, 12, -3,\ 0,\ 15,\ 3, -9,\ 0]$ and $h[n] = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$, compute the system output.
  - What does this filter do?

- Suppose we have a digital filter $h[n]$ with an unknown impulse response. We do know the system output to the follow two input signals. Determine the impulse response in terms of the two system outputs.
  - $x_1[n] = [2, 4, 2, 4] \rightarrow y_1[n]$
  - $x_2[n] = [0, 2, 1, 2] \rightarrow y_2[n]$

BIBO Stability Example

- Suppose we have a system response given by $H(z) = \frac{1}{1+z^{-2}}$. Which of the following bounded inputs would cause this system to have an unbounded output?
  - $\cos\left(\frac{\pi}{2}n\right)$
  - $\delta[n]$
  - $u[n]$
  - $e^{j\frac{\pi}{2}n}u[n]$