HKN Phys 212 Exam 2 Review Session

STEVEN KOLACZKOWSKI
KANAD SARKAR
Capacitors in Circuits

- **Series Capacitors:**
  \[ C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} \]
  - Q is the same for both capacitors

- **Parallel Capacitors:**
  \[ C_{total} = C_1 + C_2 \]
  - V is the same for both capacitors

- \( C \equiv \frac{Q}{V} \)
  \[ E = \frac{1}{2} CV^2 = \frac{1}{2} QV = \frac{1}{2} \frac{Q^2}{C} \]
Resistance, Ohm’s law, and resistors

- Ohm’s Law: $J = \sigma E$ this is usually represented as $V = IR$ in circuit analysis
  - Here $J$ is the current density, $\sigma$ is the conductivity, $I$ is the current, and $R$ is the resistance

- Resistance: a measure of an element’s opposition to charge flow $R = \frac{L}{\sigma A} = \frac{\rho L}{A}$
  - Series resistance: $R_{total} = R_1 + R_2$
    - Resistors in series share the same current ($I$)
  - Parallel resistance: $R_{total} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$
    - Resistors in parallel share the same voltage
Kirchhoff's Laws

• Kirchhoff’s Current Law (KCL): Statement of conservation of Charge
  • $\Sigma_{in} I = \Sigma_{out} I$ for any node / equipotential

• Kirchhoff’s Voltage Law (KVL): Statement of conservation of Energy
  • $\Sigma_i V_i = 0$ for any closed loop
RC Circuits

- Capacitors instantaneous reaction and time dependence.
  - At $t = 0$
  - At $t \to \infty$
- $V = IR$ only works for resistors.
  - Don’t try and apply this to capacitors and sources!!!
- Time constant ($\tau$): The fundamental time of a circuit $\tau_{RC} = RC$
  - We will have another time constant for LR and LC circuits
- Power
  - $\Sigma_i P_i = 0$ Conservation of Energy, time independent
  - $P = IV$
Magnetism

• Updated Lorentz Force: \( \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \)
  • Current Carrying Wire: \( \vec{F} = I \vec{L} \times \vec{B} \)

• Right Hand Rules:
  • 1. Thumb in direction of current, figures curl in direction of Magnetic Field
  • 2. Index figure in direction of velocity, middle figure in direction of Magnetic field, thumb in direction of force
  • 3. Figures in direction of current, thumb in direction of magnetic field.
Torque and Moments

- Torque is the rotational force applied to an object
  - \( \vec{\tau} = \vec{r} \times \vec{F} = \vec{\mu} \times \vec{B} \)

- Magnetic Moment (\( \vec{\mu} \)): Related to the amount of area and current in a system
  - \( \vec{\mu} = NI\vec{A} \)

- Potential Energy of a magnetic system: \( U = -\vec{\mu} \cdot \vec{B} \)
Biot-Savart Law and Ampere’s Law

• Biot-Savart Law: \( d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^2} \)
  
  • Since \( \vec{F} = I\vec{L} \times \vec{B} \) Force between two wires: \( F = \frac{\mu_0}{2\pi d} I_1 I_2 L \)

• Very Situational

• Ampere’s law: \( \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} \)

  • Useful in highly symmetric situations (wires and current sheets)

  • Wire: \( \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \)

  • Sheet: \( B = \frac{1}{2} \mu_0 nI \)
Electromotive Force and Faraday’s Law

- Magnetic Flux: Number of Magnetic Field Lines that penetrate a certain area
  \[ \Phi_B = \vec{B} \cdot \vec{A} = \oint \vec{B} \cdot d\vec{A} \]

- Electromotive Force: the Voltage accumulated by traveling one complete path.
  \[ \epsilon = -\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{l} \]
  - That minus sign is important!!! The induced current / field opposes a change in magnetic flux
Inductance and Inductors

• Inductance (L): A substance’s opposition to a change in current
  
  \[ L \equiv \frac{\Phi_B}{I} = \oint B \cdot dA = \frac{BA}{I} \quad \Rightarrow \quad \text{T} \cdot \text{m}^2 / \text{A} = H \]

  \[ \epsilon = -L \frac{dl}{dt} \quad U_L = \frac{1}{2} Ll^2 \]

• Solenoid: Circularly coiled wire often used as an Inductor
  
  \[ B_{\text{solenoid}} = I \mu_0 n \hat{z}, \text{ where } n \text{ is the number of coils per unit length} \]

  • Proof using Ampere’s Law

  \[ \Phi_B = \oint B_{\text{solenoid}} \cdot d\vec{S} = B_{\text{solenoid}} NA_{\text{solenoid}} = I \mu_0 nN \pi r^2 = I \mu_0 n^2 \pi r^2, \text{ where } N \text{ is the number of coils and } z \text{ is the length of the solenoid} \]

• Energy Density:
  
  \[ u_B = \frac{B^2}{2\mu_0} \quad u_E = \frac{1}{2} \epsilon_0 E^2 \]
LR Circuits

• KVL and KCL still apply! (And always will)

• \( V_L = L \frac{dl}{dt} \) \( V_R = IR \)

• Inductors will never drastically change current
  • Same as a Capacitors relation to Voltage
  • \( t = 0 \)
  • \( t \to \infty \)

• New Time Constant!!!
  • \( \tau_{LR} = \frac{L}{R} \) \( \tau_{RC} = RC \)
Exam Advice

• Know when and how to use your equation sheet
• Don’t panic, just keep on moving
• Make sure you are in the right mindset going into the exam
• Spend your time showing what you know
• DON’T CHEAT
Past Exam Questions
Which one of the choices listed on the right is an expression for the magnitude of the current $I$ in terms of $V_1$, $V_2$, $R_1$ and $R_2$?

a. $I = \frac{V_1 - 2V_2}{R_1 + 2R_2}$
b. $I = \frac{V_2}{R_1 + 2R_2}$
c. $I = \frac{V_2}{3R_1 + 2R_2}$
d. $I = \frac{V_1 - V_2}{3R_1 + 2R_2}$
e. $I = \frac{V_1}{R_1 + 2R_2}$

The potential at point A is _________ the potential at point B.

(a) lower than
(b) the same as
(c) higher than
Three resistors \((R_1 = 15 \, \Omega, \, R_2 = 30 \, \Omega, \, R_3 = 60 \, \Omega)\) and two capacitors \((C_1 = C_2 = 2.4 \times 10^{-5} \, \text{F})\) are connected to a battery with a potential difference of 18 volts as shown in the figure. The capacitors are initially uncharged. At time \(t=0\) the switch is moved to position \(a\).

1) What is the current through resistor \(R_1\) immediately after the switch is moved to position \(a\)?

2) What is the charge on capacitor \(C_1\) after the switch has been in position \(a\) for a long time?

3) What is the time constant associated with discharging the capacitors when the switch has been moved to position \(b\)?
A long solid conducting cylinder of radius $a = 0.4 \text{ m}$ is centered on the z-axis $(x=0, y=0)$ and carries a current $I_1 = 11 \text{ A}$. This current is uniformly distributed throughout the cylinder. A long wire is located a distance $b = 1.2 \text{ m}$ to the right of the cylinder $(x=0, y=1.2)$. Initially this wire carries no current ($I_2 = 0$).

1) What is the magnitude of the magnetic field a distance $(r = 0.267 \text{ m})$ from the center of the cylinder?

2) What value of current in the wire, will result in the magnetic field due to the two currents, being zero on the y axis at $(x=0, y=1.04 \text{ m}, z=0)$?
Two fixed conductors are connected by a resistor $R=14 \, \Omega$. The two fixed conductors are separated by $L = 1.9 \, \text{m}$ and lie horizontally. A conducting bar of mass $m=0.4 \, \text{kg}$ slides on them at a constant speed $v$, producing a current of $0.85 \, \text{amps}$. A magnetic field (shown by the black dots in the figure) with magnitude $5.2 \, \text{T}$ points out of the page.

1) In which direction does the induced current flow through the moving conducting bar when the bar is sliding in the direction shown?

2) How fast is the bar moving?
   - $v = 1.2 \, \text{m/s}$
   - $v = 0.086 \, \text{m/s}$
   - $v = 2.29 \, \text{m/s}$

3) What is the magnitude of the magnetic force on the bar moving at this speed?
   - $F = 0.48 \, \text{N}$
   - $F = 8.4 \, \text{N}$
   - $F = 0.311 \, \text{N}$
A circuit is constructed with three resistors and one inductor as shown. The values for the resistors are: $R_1 = 2 \, \Omega$ and $R_2 = R_3 = 4 \, \Omega$. The battery voltage is $V = 12 \, \text{V}$. The switch $S$ is initially open.

1) After the switch has been opened a long time, it is closed. What is the magnitude of the voltage across $L$ immediately after the switch is closed.

2) After the switch has been closed a long time it is opened again. What is the magnitude of the current through $R_2$ immediately after the switch is opened?

3) After the switch is opened, it takes the current a time $6 \, \mu\text{s}$ to drop to $1/e$ of its initial value. What is the value of the inductance of the circuit?
2) The loop is allowed to relax to its lowest energy position. What is the work done on the loop by the magnetic field?

\begin{align*}
&IB(ac + bd) \\
&2IB(a + b + c + d) \\
&IB(a + b + c + d) \\
&-IB(a + b + c + d) \\
&-IB(ac + bd)
\end{align*}

3) Which of these sketches best represents the potential energy as a function of the rotation of the wire loop? (The angle \( \theta = 0 \) initially when the loop is in the x-y plane, and increases with clockwise rotation around the y-axis, as depicted at right).

A wire loop is attached to an axis (dotted line in Figure) about which it can freely rotate. There is a constant magnetic field and a current flowing counter-clockwise in the wire as shown in the image. The length of the line segments are \( a, b, c, \) and \( d \) as indicated.

1) What is the net torque about the axis on the wire loop?

\begin{align*}
&-IB(ac + bd)\hat{y} \\
&IB(ac + bd)\hat{y} \\
&-IB(a + b + c + d)\hat{y} \\
&IB(a + b + c + d)\hat{y} \\
&0
\end{align*}