HKN CS 374 Midterm 1 Review

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Midterm topics

It's all about recognizing sets of strings!

1. String Induction
2. Regular languages
   a. DFA
   b. NFA
   c. Regular expressions
3. Context-free languages
   a. Context-free grammar

String Induction

- Proof standards are strictest here - stick to the definitions!
- Be careful of the order of the if-else branches

$$|w| := \begin{cases} 0 & \text{if } w = \varepsilon \\ 1 + |x| & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

$$w \bullet z := \begin{cases} z & \text{if } w = \varepsilon \\ a \cdot (x \bullet z) & \text{if } w = ax \text{ for some symbol } a \text{ and some string } x \end{cases}$$

Lemma 1: $w \bullet \varepsilon = w$ for all strings w.

Lemma 2: $|w \bullet x| = |w| + |x|$ for all strings w and x.

Lemma 3: $(w \bullet x) \bullet y = w \bullet (x \bullet y)$ for all strings w, x, and y.

Language Definitions

- A DFA *recognizes* (or accepts) a language L iff every string in L is accepted by the DFA, and every string not in L is rejected by the DFA.
    - Likewise for NFAs, regular expressions
- A context-free language L is *generated by* a CFG iff (almost copy and paste from above)

Regular Languages & Expressions

- Formally, one of
  a. The empty set
  b. The set containing one string
  c. The union, concatenation, or Kleene closure of regular language(s)
- Additionally, closed under intersection, set difference, reversal, homomorphisms, inverse homomorphisms
- Regular expressions follow the structure of the formal definition

Deterministic Finite Automata

- Formally,
  - An arbitrary finite set Σ, called the ***input alphabet***.
  - Another arbitrary finite set Q, whose elements are called ***states***.
  - An arbitrary ***transition*** function $\delta : Q \times \Sigma \to Q$.
  - A ***start state*** $s \in Q$.
  - A subset $A \subseteq Q$ of ***accepting states***.

- In drawings, Q is the set of circles, δ is the set of arrows, s is the circle with an arrow leading into it without any source, and A is the set of doubly-circled circles

Non-deterministic Finite Automata

- Formally,

Definition

A non-deterministic finite automata (NFA) $N = (Q, \Sigma, \delta, s, A)$ is a five tuple where

- Q is a finite set whose elements are called states,
- Σ is a finite set called the input alphabet,
- $\delta : Q \times \Sigma \cup \{\varepsilon\} \to \mathcal{P}(Q)$ is the transition function (here $\mathcal{P}(Q)$ is the power set of Q),
- $s \in Q$ is the start state,
- $A \subseteq Q$ is the set of accepting/final states.

- Drawn similarly like DFAs, but with the extra epsilon transitions
- An NFA accepts a string s can reach *at least one* of the accepting states

Regular Languages: Useful Theorems

Theorem (DFA=NFA=ε-NFA)

A language is accepted by a deterministic finite automaton if and only if it is accepted by a non-deterministic finite automaton.

Theorem (REGEX=DFA)

A language is accepted by a deterministic finite automaton if and only if it is accepted by a regular expression.

Source: CSCI

Is it regular?

- First, determine if the language definition can be simplified
- Then, check if you need an unbounded counter
- If so, try to create a fooling set
  - If possible, choose the simplest prefixes you can find
  - Be careful of generalized string splits - a string belongs to a presented language if *any* permitted combo of strings satisfies the conditions within the set condition
  - Use letter changes to guard against these splits
- If not, try to create a DFA/NFA/Regex

Regular Expressions: Examples

Write regular expressions for the following languages over the alphabet $\Sigma = \{a, b\}$:

(a) All strings that do not end with aa
(b) All strings that contain an even number of b's
(c) All strings which do not contain ba

Source: [Stanford]

Regular Expressions: Examples

Write regular expressions for the following languages over the alphabet Σ = {a, b}:

(a) All strings that do not end with aa

$\in + a + b + (a + b)^{*} (ab + ba + bb)$

(b) All strings that contain an even number of b's

$a^{*} (ba^{*} ba^{*})^{*}$

(c) All strings which do not contain ba

$a^{*} b^{*}$

Source: Stanford

Regular Languages: Example

Is the language $\{ xyx^r \mid x, y \in \{0,1\}^+ \}$ regular?

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Regular Languages: Example

Is the language $\{ xyx^r \mid x, y \in \{0,1\}^+ \}$ regular?

Answer: Yes - equivalent to $0(0+1)^+0 + 1(0+1)^+1$

Is the language $L = \{ xyxyxy \mid x, y \in \{0,1\}^+ \}$ regular?

Answer: No - use 1 to indicate end of string xy in fooling set

Fooling set: $\mathbf{F} = \{0^n1^n \mid n >= 0\}$

Suffix ($\mathbf{z}$) : $0^n1^n0^n1^n$

$\mathbf{xz} \in L$ but $\mathbf{yz} \notin L$

$\mathbf{x} = 0^n1^n$

$\mathbf{y} = 0^i1^i \quad i \neq n$

Deterministic Finite Automata: Examples

Draw DFAs for the following languages over the alphabet $\Sigma = \{a, b\}$:

(a) All strings that do not end with aa
(b) All strings that contain an even number of b's
(c) All strings which do not contain ba

Source: Stanford

Deterministic Finite Automata: Examples

Draw DFAs for the following languages over the alphabet Σ = {a, b}:

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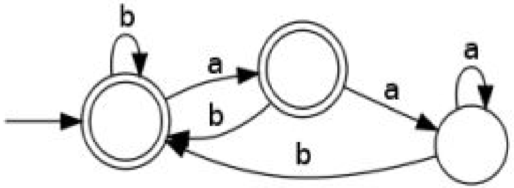
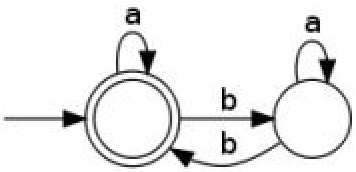


(a)



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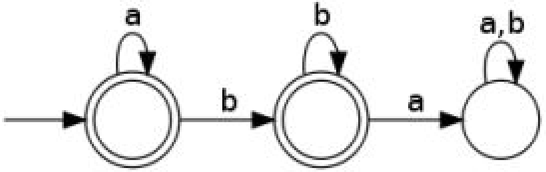

(c)

Source: [Stanford]

Context-free Grammars and Languages

- Formally,
  - A finite set Σ, whose elements are called ***symbols*** or ***terminals***.
  - A finite set Γ disjoint from Σ, whose elements are called ***non-terminals***.
  - A finite set R of ***production rules*** of the form $A \to w$, where $A \in \Gamma$ is a non-terminal and $w \in (\Sigma \cup \Gamma)^*$ is a string of symbols and variables.
  - A ***starting*** non-terminal, typically denoted S.
- *Not* closed under intersection or set difference
- Design:
  a. Pair letters - identify their positions and guarantee that substrings hold certain properties
  b. Drop letters - useful for inequalities or early termination
  c. Cumulative counting - does the difference operator Δ hit zero?

Context-free Grammars: Examples

Construct context-free grammars for each of the following languages:

a) $\{a^m b^n \mid m >= n\}$
b) $\{a^m b^n c^p d^q \mid m + n = p + q\}$
c) $\{w \in (a+b)^* \mid$ w has twice as many b's as a's$\}$
d) $\{uawb \mid u, w \in (a+b)^*, |u| = |w|\}$

Source: [UT-Austin]

Context-free Grammars: Examples

Construct context-free grammars for each of the following languages:

a) $\{a^m b^n \mid m >= n\}$

Answer: S → aSb | aCb | ε, C → aC | ε

b) $\{a^m b^n c^p d^q \mid m + n = p + q\}$

Answer: S → aSd | T | U, T → aTc | V, U → bUd | V, V → bVc | ϵ

c) {w ∈ (a+b)* | w has twice as many b's as a's}

Answer: S → SaSbSbS | SbSaSbS | SbSbSaS | ϵ

d) {uawb | u, w ∈ (a+b)*, |u| = |w|}

Answer: S → Tb, T → aTa | aTb | bTa | bTb | a

Source: UT-Austin

Language Transformation

Let L_1 and L_2 be regular languages that share a common alphabet $\Sigma = \{0, 1\}$. Define the bitwise-xor of L_1 and L_2 as

$$xor(L_1, L_2) = \{xor(x, y) \mid x \in L_1 \text{ and } y \in L_2\},$$

where $xor(x, y)$ is the bitwise-xor of two binary strings x and y. Prove that $xor(L_1, L_2)$ is regular.

Language Transformation

$$Q_3 = Q_1 \times Q_2$$
$$\delta_3((q_1, q_2), 0) = \{(\delta_1(q_1, 0), \delta_2(q_2, 0)), (\delta_1(q_1, 1), \delta_2(q_2, 1))\}$$
$$\delta_3((q_1, q_2), 1) = \{(\delta_1(q_1, 0), \delta_2(q_2, 1)), (\delta_1(q_1, 1), \delta_2(q_2, 0))\}$$
$$s_3 = (s_1, s_2)$$
$$A_3 = A_1 \times A_2$$

True/False Examples

G.4 Let M be an **NFA** over the alphabet Σ. Let M' be identical to M, except that accepting states in M are non-accepting in M' and vice versa. Each string in Σ^* is accepted by exactly one of M and M'.

True/False Examples

B.36 If $L \subseteq L'$ and L is not regular, then L' is not regular.

True/False Examples

H.9 For every context-free language L, the language $\{w^R \mid w \in L\}$ is also context-free.

True/False Examples

Let $S = \{L_1, L_2, L_3, \ldots\}$ be an *infinite* set of regular languages. Then, the union of all of the elements of S is a regular language.

True/False Examples

B.12 For all languages L, if L^* is regular, then L is regular.

True/False Examples

C.15 For all context-free languages A, B, and C, the language $(A \cup B)^* \bullet C$ is also context-free.