HKN ECE 310 Quiz 6 Review Session

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Topics

- Digital Rate Conversion: Upsampling, Downsampling
- D/A: Review of Ideal and ZOH, Upsampled D/A
- FFT: Decimation in Frequency and Time
- Circular Convolution, Linear Convolution using FFT
Upsampling

- If we upsample by $L$, we will interpolate $L - 1$ zeros between each sample
  - $y[n] = x[n/L]$ if $n \mod L = 0$
  - 0 else

- What happens in the frequency domain?
  - Think about what happens when we oversample and signal, i.e. above Nyquist?

- What does the frequency response look like after upampling?
  - Shrink x-axis by factor of $L$

- What should $G_D(\omega)$ be in order to obtain a desirable frequency response?
  - Remove extra copies and correct amplitude for conservation of energy
Review: Ideal D/A

- Want $Y\downarrow a(\Omega) = Y\downarrow d(\Omega T)$, but we want to only take one copy of the DTFT
  - Thus, we should low-pass filter from $\Omega = \pm \pi/T$ (domain of the central copy of the DTFT)
- Inverse CTFT of $Trect(\Omega/2\pi/T) = \text{sinc}(\pi/T \cdot t)$
- Remember that multiplication in the frequency domain is convolution in the time domain
  - Thus, $y\downarrow a(t) = \sum_{n=-\infty}^{\infty} y[n] \text{sinc}[\pi/T \cdot (t-nT)]$, where $y\downarrow n$ is obtained by multiplying $y[n]$ by an impulse train
Review: Zero-Order Hold

- Ideal D/A is not practical because generating delta impulses is not achievable
- Zero-Order Hold (ZOH) gives us a suitable approximation to the Ideal D/A
- The ZOH multiplies each sample by a rectangular pulse of width T (our sampling rate)
  - Thus, \( y(t) = \sum_{n=-\infty}^{\infty} y(n) p(t-nT) \) where \( p(t) \) is the rectangular pulse provided by the ZOH
- \( F(t) \) is an analog filter that corrects the distortion presented by the ZOH

![Diagram](image-url)

In manufacturer’s catalog just the ZOH may be called a D/A
Upsampled D/A

- Upsampling prior to D/A conversion can make recovery simpler
  - i.e. Compensator $F_a(\Omega)$ can be simpler to implement
- Upsampling effectively increases our sampling frequency, thus our ZOH pulse can be narrower and give us a better staircase approximation
- This ‘smoother staircase’ will be easier to rectify with the compensator
  - i.e. the transition bandwidth will be larger
- In the frequency domain, we see the frequency axis compress by $L$; however, the analog frequencies upon recovery do not change!
Downsampling

- If we downsample by $D$, we keep every $D^{th}$ sample (decimate the rest)
  - $y[n] = x(Dn)$
- What happens in the frequency domain?
  - Frequency response stretches by a factor of $D$
  - Amplitude reduces by a factor of $D$ (think conservation of energy)
- Anti-aliasing filter prevents downsampling from aliasing our signal
  - $A(\omega) = \text{LPF with } \omega \downarrow c = \pi/D$
Fast Fourier Transform

- Computational efficient implementation of the DFT
  - Ordinary DFT requires $N^2$ multiplies and $N(N - 1)$ adds
  - FFT requires only $O(M\log_2 N)$ computations
- Two main forms of FFT
  - Decimation in Time
  - Decimation in Frequency
- We will consider the Radix 2 FFT, but other Radices may be used
- Use Butterfly Structures to represent multiplies and adds
Decimation in Time

- Divide sequence into two groups
  - $y[n] = x[2n]$ and $z[n] = x[2n + 1]$
  - Before we continue, remember that $\downarrow N = e^{-j2\pi/N}$
- We can form the first half of the DFT from these two sequences
  - $X[k] = \sum n y[n] \downarrow 2n W \downarrow N/2 k n + x[n] \downarrow 2n + 1 W \downarrow N k (2n + 1) = \sum n y[n] \downarrow N/2 k n + W \downarrow N k \sum n z[n] \downarrow N/2 k n$ for $0 \leq k \leq N/2 - 1$
- And the second half…
  - $X[k] = \sum n y[n] \downarrow N/2 k n - W \downarrow N k \sum n z[n] \downarrow N/2 k n$ for $N/2 \leq k \leq N$
- Continually halve the sequences until you reach size 2
Decimation in Frequency

- Reverse of Decimation of Time
- Compute points over entire DFT in even and odd groups
  - DIT divides even and odd groups then computes points over smaller groups
- Even k’s: \( X_{2k} = \sum_{n=0}^{N/2-1} (x_{2n} + x_{2n+N/2}) W^{n/N} W_{2k}^{n} \)
- Odd k’s: \( X_{2k+1} = \sum_{n=0}^{N/2-1} (x_{2n} - x_{2n+N/2}) W^{n/N} W_{2k+1}^{n} \)
Butterfly Structure Exercise

- Draw the butterfly structure of a length 8 Decimation in Time FFT for $x_n$
Butterfly Structure Exercise

- Draw the butterfly structure of a length 8 Decimation in Time FFT for $x_n$
Butterfly Structure Exercise

- Decimation in Frequency is reverse of Decimation in Time
Fast Linear Convolution

- In order to obtain system response, we can multiply DFTs and take inverse DFT
  - Be careful, this is not the same as convolution in time, but rather cyclic convolution in time
- Therefore, in order to perform linear convolution from DFTs, we must first zero pad signals in order to make wrap-around terms go to zero
- If $x_n$ has length $N$ and $h_n$ has length $M$, then the resulting convolution is of length $N+M-1$
  - We must pad $x_n$ with $M-1$ zeros
  - $h_n$ with $N-1$ zeros
- This will allow multiplication of FFTs to produce linear convolution result from cyclic convolution
- Note: we also typically do extra padding so that the signals are of power of 2 length to optimize the FFT
References

✧ Chapters 13 and 14 in Singer’s Textbook
✧ Overlap Add and Overlap Save Stuff @ Chapter 14 Page 16+