HKN ECE 310 Exam 1 Review Session

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Topics

- DTFT
- DFT
- Windowing and Spectral Analysis
- LSIC Systems
- Sampling
- Convolution
- Impulse Response
Discrete Time Fourier Transform

\[ X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \]

\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega)e^{j\omega n} d\omega \]

- Important Properties:
  - Periodicity!
  - Linearity
  - Symmetries (Magnitude, angle, real part, imaginary part)
  - Time shift and modulation
  - Product of signals and convolution
  - Parseval’s Relation
  - Know your geometric series sums!
Discrete Fourier Transform

\[ X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi kn}{N}} \]

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{\frac{j2\pi kn}{N}} \]

✱ What is the relationship between the DTFT and the DFT?

✱ \( \omega_k = \frac{2\pi k}{N} \)
Discrete Fourier Transform Properties

- *Circular* shift
- *Circular* modulation
- *Circular* convolution
- Why are these properties circular?
- Parseval’s Relation
DTFT and DFT Examples

- Suppose we have a signal $x[n]$ with DTFT $X_d(\omega)$. Find the DTFT for the following signals in terms of $X_d(\omega)$
  - $y[n] = x[n]\cos(\omega_0 n)$
  - $z[n] = x[n - n_0] + x[n + n_0]$

- Drawing Example! (Should have come to the review session.)

- Suppose we have a signal $x[n] = [1, 2, 3, 4, 5, 6]$ with DFT $X[k]$. Find the matching signal or DFT that corresponds to the following DFTs in terms of $x[n]$ and $X[k]$
  - $Y[k] = X[k]e^{-jn_k}$
  - $z[n] = [1, -2, 3, -4, 5, -6]$
Windowing and Spectral Analysis

◊ Signals cannot go to infinity
  ◊ Therefore, we need to window

◊ There are many different windows
  ◊ Rectangular (boxcar)
  ◊ Hamming
  ◊ Hanning
  ◊ Triangular
  ◊ Kaiser

◊ More on advantages/disadvantages later
Windowing and Spectral Analysis

- What happens when we dictate that $x[n] = \cos(\omega_0 n)$ is of finite duration $N$?
  - Derivation on page 54 of textbook

- Spectral Analysis: Resolving different sinusoidal frequency components in a signal

- The DTFT of a finite sinusoidal signal has main lobe width of $\frac{4\pi}{N}$ where $N$ is the # of samples in the signal

- Resolution can be defined in different ways
  - Full lobe resolution vs. Half lobe resolution
Full-Lobe vs. Half-Lobe Resolution

❖ Suppose we represent a cosinusoid as $A\cos(\Omega T)$
❖ The lobe centers of two cosinusoids will be located at $\Omega_0 T$ and $\Omega_1 T$
   ❖ Remember that the half-width of each lobe is $\frac{2\pi}{N}$
❖ Full-Lobe
   ❖ To prevent crossover: $\Omega_0 T + \frac{2\pi}{N} < \Omega_1 T - \frac{2\pi}{N}$ \rightarrow $\Omega_1 - \Omega_0 > \frac{4\pi}{NT}$
❖ Half-Lobe
   ❖ $\Omega_0 T < \Omega_1 T - \frac{2\pi}{N}$ \rightarrow $\Omega_1 - \Omega_0 > \frac{2\pi}{NT}$
Zero-Padding

◊ We can improve the resolution of the DFT simply by adding zeros to the end of the signal

◊ This doesn’t change the frequency content of the DTFT!

◊ Instead, it increases the number of samples the DFT takes of the DTFT

◊ This can be used to improve spectral analysis
Window Comparisons

- Rectangular (boxcar)
  - Maintains width of the main lobe, thus better resolution
  - Poor side lobe attenuation, can lead to resolution errors
- Hamming
  - Doubles the width of the main lobe, thus poorer resolution
  - Greatly reduces side lobes, prevents mistaking side lobes as main lobes of other frequencies
- Kaiser
  - Optimal
LSIC Systems

◊ **Linearity**
  ◊ Satisfy Homogeneity and Additivity
  ◊ Can be summarized by Superposition
    ◊ If $x_1[n] \rightarrow y_1[n]$ and $x_2[n] \rightarrow y_2[n]$, then $ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$  

◊ **Shift Invariance**
  ◊ If $x[n] \rightarrow y[n]$, then $x[n - n_0] \rightarrow y[n - n_0] \ \forall \ n_0$ and $x[n]$  

◊ **Causality**
  ◊ Output cannot depend on future input values
LSIC Examples

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<tbody>
<tr>
<td></td>
<td>For the following systems, determine whether it is linear, shift-invariant, and causal</td>
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<tr>
<td>y[n] = x^2[n]</td>
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<td>y[n] = x[</td>
<td>n</td>
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<tr>
<td>y[n] = x[n]cos(\omega_0 n)</td>
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Sampling

- For ideal A/D sampling, the relationship between the CTFT and DTFT is as follows

\[ X_d(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a \left( \frac{\omega + 2\pi k}{T} \right) \]

- Why is there a \(2\pi k\)?
- Why is there a \(\frac{1}{T}\) factor?

\[ \omega_d = \Omega_a T \]

- Nyquist Criterion: \(\frac{1}{T} > 2B\)
Convolution

\[ y[n] = \sum_{m=\infty}^{\infty} x[m] h[n - m] = \sum_{m=\infty}^{\infty} h[m] x[n - m] \]

- System must be:
  - Linear
  - Shift Invariant

- Popularly done graphically
- Be comfortable doing it algebraically
Impulse Response

◊ System output to an $x[n] = \delta[n]$ input

◊ $y[n] = x[n] * h[n]$

◊ $Y_d(\omega) = X_d(\omega)H_d(\omega)$
Impulse Response and Convolution Examples

- Given \( x[n] = [6, 12, -3, 0, 15, 3, -9, 0] \) and \( h[n] = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}] \), compute the system output.
  - What does this filter do?

- Suppose we have a digital filter \( h[n] \) with an unknown impulse response. We do know the system output to the follow two input signals. Determine the impulse response in terms of the two system outputs.
  - \( x_1[n] = [2, 4, 2, 4] \to y_1[n] \)
  - \( x_2[n] = [0, 2, 1, 2] \to y_2[n] \)