ECE 329 Exam 3

HKN Review

Fall 2017

Poynting Theorem and Poynting Flux

$$\mathbf{S} \equiv \mathbf{E} \times \mathbf{H}$$

$$\frac{\mathrm{V}}{\mathrm{m}}\frac{\mathrm{A}}{\mathrm{m}} = \frac{\mathrm{W}}{\mathrm{m}^2} = \frac{\mathrm{J/s}}{\mathrm{m}^2}$$

Poynting thm:

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\epsilon\mathbf{E}\cdot\mathbf{E} + \tfrac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}\right) + \nabla\cdot(\mathbf{E}\times\mathbf{H}) + \mathbf{J}\cdot\mathbf{E} = 0$$

- **Power per unit area!**

Monochromatic Wave Solutions and Phasor Notation

$$v = \frac{\omega}{\beta}$$

$$v = \frac{\lambda}{T} = \lambda f$$

$$\lambda \equiv \frac{2\pi}{\beta} \quad \text{Wavelength}$$

$$T = \frac{2\pi}{\omega} \equiv \frac{1}{f} \quad \text{Waveperiod.}$$

Field	Phasor	Comment
$\mathbf{E} = \cos(\omega t + \beta y)\,\hat{z}$	$\tilde{\mathbf{E}} = e^{j\beta y}\,\hat{z}$	z-polarized wave propagating in $-y$ direction
	$\tilde{\mathbf{H}} = -\frac{e^{j\beta y}}{\eta}\,\hat{x}$	magnetic phasor that accompanies $\tilde{\mathbf{E}}$ above
$\mathbf{H} = \sin(\omega t - \beta z)\,\hat{y}$	$\tilde{\mathbf{H}} = -je^{-j\beta z}\,\hat{y}$	wave propagating in $+z$ direction
$\mathbf{E} = \eta\sin(\omega t - \beta z)\,\hat{x}$	$\tilde{\mathbf{E}} = -j\eta e^{-j\beta z}\,\hat{x}$	electric field phasor of $\tilde{\mathbf{H}}$ above which is an x-polarized field (see the right column)

Propagation in Various Media

$$\bar{\gamma} = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right)^{1/2}$$

attenuation

$$e^{-\alpha z}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right)^{1/2}$$

velocity

$$v_p = \omega/\beta$$

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

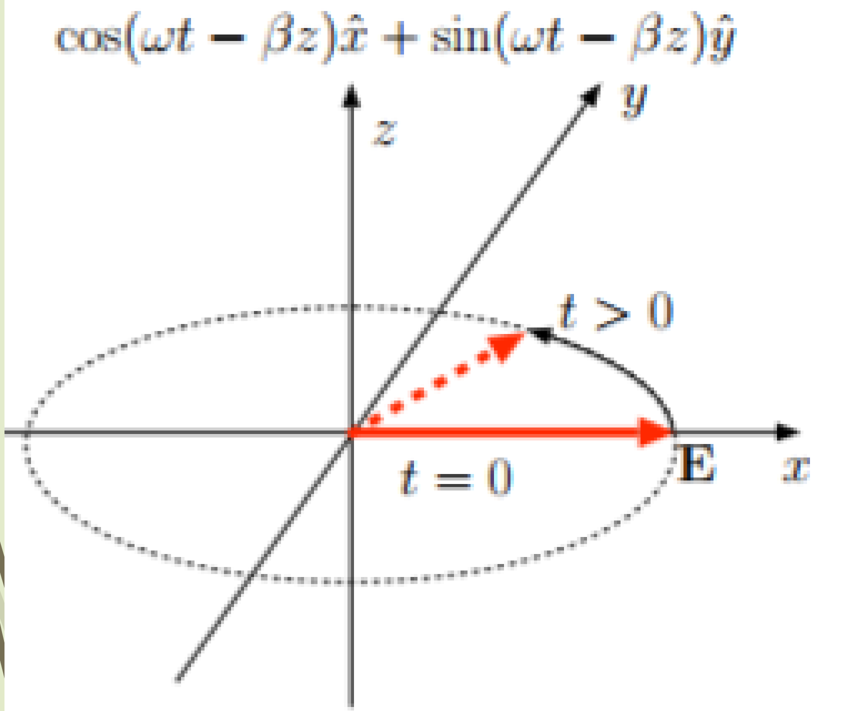
Phasor Form of Maxwell's Equations, Fields in Various Media

$$\begin{aligned}\nabla \cdot \tilde{\mathbf{E}} &= 0\\ \nabla \cdot \tilde{\mathbf{H}} &= 0\\ \nabla \times \tilde{\mathbf{E}} &= -j\omega\mu\tilde{\mathbf{H}}\\ \nabla \times \tilde{\mathbf{H}} &= j\omega\epsilon\tilde{\mathbf{E}}\end{aligned}$$

	Condition	β	α	$\lvert\eta\rvert$	τ	$\lambda = \frac{2\pi}{\beta}$	$\delta = \frac{1}{\alpha}$
Perfect dielectric	$\sigma = 0$	$\omega\sqrt{\epsilon\mu}$	0	$\sqrt{\frac{\mu}{\epsilon}}$	0	$\frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	∞
Imperfect dielectric	$\frac{\sigma}{\omega\epsilon} \ll 1$	$\sim \omega\sqrt{\epsilon\mu}$	$\beta\frac{1}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\sim \sqrt{\frac{\mu}{\epsilon}}$	$\sim \frac{\sigma}{2\omega\epsilon}$	$\sim \frac{2\pi}{\omega\sqrt{\epsilon\mu}}$	$\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$
Good conductor	$\frac{\sigma}{\omega\epsilon} \gg 1$	$\sim \sqrt{\pi f\mu\sigma}$	$\sim \sqrt{\pi f\mu\sigma}$	$\sqrt{\frac{\omega\mu}{\sigma}}$	45°	$\sim \frac{2\pi}{\sqrt{\pi f\mu\sigma}}$	$\sim \frac{1}{\sqrt{\pi f\mu\sigma}}$
Perfect conductor	$\sigma = \infty$	∞	∞	0	-	0	0

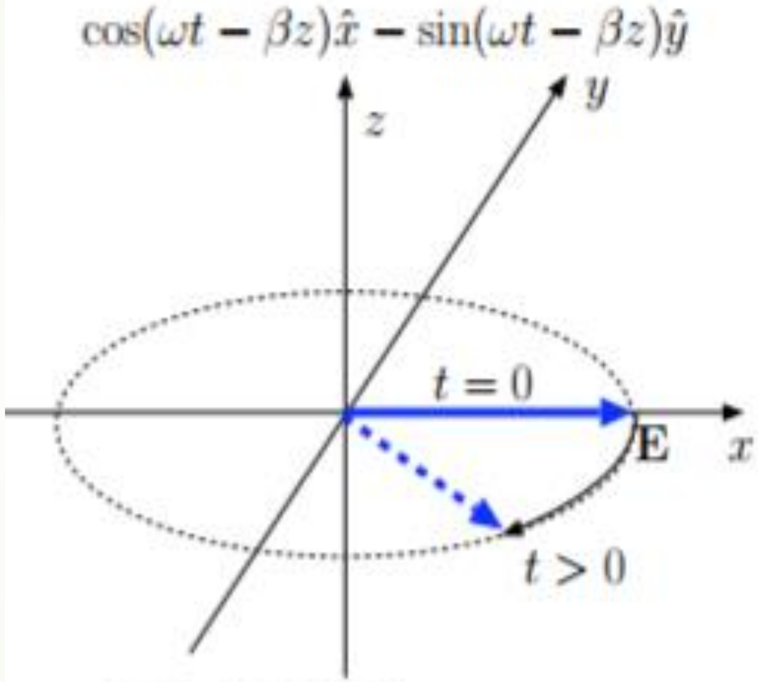
Circular Polarization

The rotation frequency is also the wave frequency.



RIGHT CIRCULAR

Or (Lead) x (Lag)



LEFT CIRCULAR

When left-hand thumb is pointed along propagation direction z the fingers curl in the rotation direction of the field vector.

Wave Reflections

$$\hat{n} \cdot (\mathbf{D}^+ - \mathbf{D}^-) = \rho_s$$
$$\hat{n} \cdot (\mathbf{B}^+ - \mathbf{B}^-) = 0$$
$$\hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) = 0$$
$$\hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) = \mathbf{J}_s$$

$$\tilde{\mathbf{E}}_i = \hat{x} E_o e^{-j\beta_1 z},$$
$$\tilde{\mathbf{H}}_i = \hat{y} \frac{E_o}{\eta_1} e^{-j\beta_1 z},$$
$$\tilde{\mathbf{E}}_r = \hat{x} \Gamma E_o e^{j\beta_1 z},$$
$$\tilde{\mathbf{H}}_r = -\hat{y} \frac{\Gamma E_o}{\eta_1} e^{j\beta_1 z},$$
$$\tilde{\mathbf{E}}_t = \hat{x} \tau E_o e^{-\gamma_2 z},$$
$$\tilde{\mathbf{H}}_t = \hat{y} \frac{\tau E_o}{\eta_2} e^{-\gamma_2 z}.$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$1 + \Gamma = \tau$$
$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Guided TM Waves, Beginning of Transmission Lines

$$-\frac{\partial V}{\partial z} = \mathcal{L}\frac{\partial I}{\partial t}$$
$$-\frac{\partial I}{\partial z} = \mathcal{C}\frac{\partial V}{\partial t}$$

where

$$\mathcal{C} = \epsilon\mathbf{GF}, \ \mathcal{L} = \frac{\mu}{\mathbf{GF}},$$

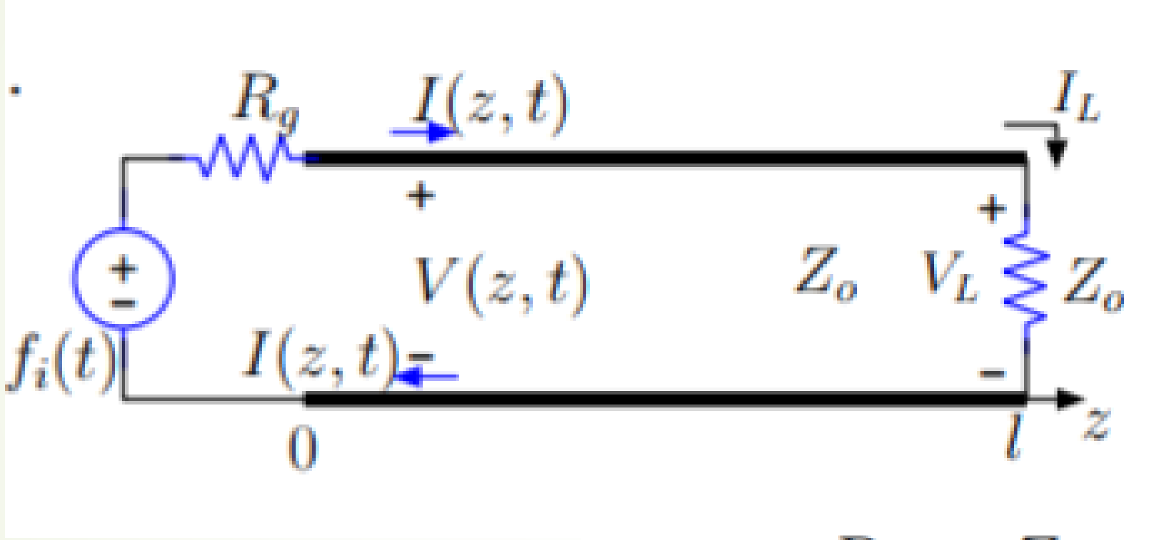
with "geometrical factor"

$$\begin{aligned} \mathbf{GF} &= \frac{W}{d} \ \textbf{parallel-plate} \\ &= \frac{2\pi}{\ln\frac{b}{a}} \ \textbf{coax} \\ &= \frac{\pi}{\cosh^{-1}\frac{D}{2a}} \ \textbf{twin-lead} \end{aligned}$$

$$v = \frac{1}{\sqrt{\mathcal{LC}}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$Z_o = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \frac{1}{\mathrm{GF}}\sqrt{\frac{\mu}{\epsilon}}.$$

Distributed Circuits (Reflections and Transmissions in Circuits)



$\tau_g = \frac{Z_o}{R_g + Z_o}$ called an **injection coefficient.**

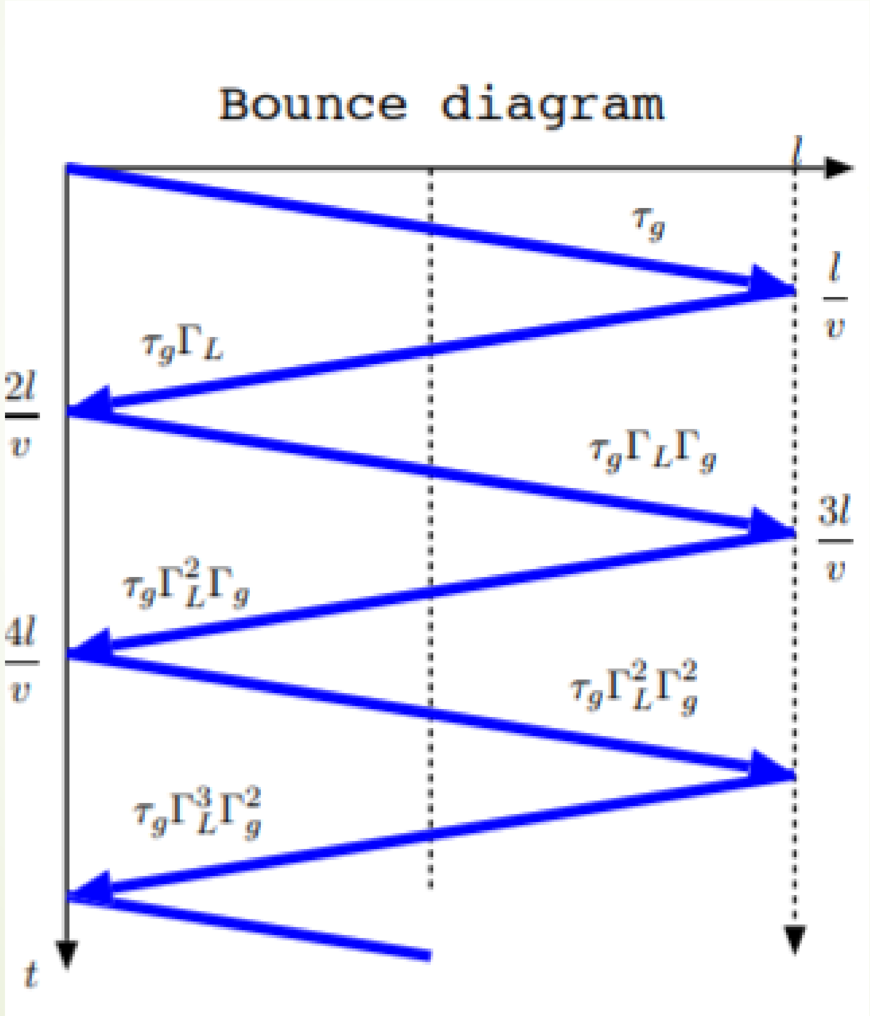
$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}. \quad \Gamma_g = \frac{R_g - Z_o}{R_g + Z_o}$$

$$V(z,t) = \tau_g \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v})$$
$$+\tau_g \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v})$$

$$I(z,t) = \frac{\tau_g}{Z_o} \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t - \frac{z}{v} - n\frac{2\ell}{v})$$
$$-\frac{\tau_g}{Z_o} \Gamma_L \sum_{n=0}^{\infty} (\Gamma_L \Gamma_g)^n \delta(t + \frac{z}{v} - (n+1)\frac{2\ell}{v}).$$

Bounce Diagrams

- We'll do examples graphically, but:



Examples (we did the Fall 2016 Exam 3 in the review for those with notes!)