HKN ECE 310 Quiz 2 Review Session

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Discrete Time Fourier Transform

\[ X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \]
\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_d(\omega) e^{j\omega n} d\omega \]

Important Properties:
- Periodicity!
- Linearity
- Symmetries (Magnitude, angle, real part, imaginary part)
- Time shift and modulation
- Product of signals and convolution
- Parseval’s Relation
- Know your geometric series sums!
Discrete Fourier Transform

\[ X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N} \]

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} \]

What is the relationship between the DTFT and the DFT?

\[ \omega_k = \frac{2\pi k}{N} \]
Discrete Fourier Transform Properties

- Circular shift
- Circular modulation
- Circular convolution
- Parseval’s Relation
Windowing and Spectral Analysis

- Signals cannot go to infinity
  - Therefore, we need to window

- There are many different windows
  - Rectangular (boxcar)
  - Hamming
  - Hanning
  - Triangular
  - Kaiser

- More on advantages/disadvantages later
Windowing and Spectral Analysis

◊ What happens when we dictate that $x[n] = \cos(\omega_0 n)$ is of finite duration $N$?
  ◊ Derivation on page 54 of textbook

◊ Spectral Analysis: Resolving different sinusoidal frequency components in a signal

◊ The DTFT of a finite sinusoidal signal has main lobe width of $\frac{4\pi}{N}$ where $N$ is the # of samples in the signal

◊ Resolution can be defined in different ways
  ◊ Full lobe resolution vs. Half lobe resolution
Full-Lobe vs. Half-Lobe Resolution

✧ Suppose we represent a cosinusoid as $Acos(\Omega T)$

✧ The lobe centers of two cosinusoids will be located at $\Omega_0 T$ and $\Omega_1 T$
  
  ✧ Remember that the half-width of each lobe is $\frac{2\pi}{N}$

✧ Full-Lobe
  
  ✧ To prevent crossover: $\Omega_0 T + \frac{2\pi}{N} < \Omega_1 T - \frac{2\pi}{N}$ $\Rightarrow$ $\Omega_1 - \Omega_0 > \frac{4\pi}{NT}$

✧ Half-Lobe
  
  ✧ $\Omega_0 T < \Omega_1 T - \frac{2\pi}{N}$ $\Rightarrow$ $\Omega_1 - \Omega_0 > \frac{2\pi}{NT}$
Zero-Padding

- We can improve the resolution of the DFT simply by adding zeros to the end of the signal.
- This doesn’t change the frequency content of the DTFT!
- Instead, it increases the number of samples the DFT takes of the DTFT.
- This can be used to improve spectral analysis.
Window Comparisons

- Rectangular (boxcar)
  - Maintains width of the main lobe, thus better resolution
  - Poor side lobe attenuation, can lead to resolution errors
- Hamming
  - Doubles the width of the main lobe, thus poorer resolution
  - Greatly reduces side lobes, prevents mistaking side lobes as main lobes of other frequencies
- Kaiser
  - Optimal