STABLE STATE, TRANSIENT, ZERO STATE, ZERO INPUT

• Steady State
  • Does not go to 0 for \( t \to \infty \), \( \lim_{t \to \infty} f(t) = f_{ss}(t) \)
  • Examples: Constants, Sinusoids

• Transient
  • Goes to zero over time, \( \lim_{t \to \infty} f_{tr}(t) = 0 \)

Given ODE \( \frac{dy}{dt} + y(t) = f(t) \)

• Zero State
  • Set \( y(t_0) = 0 \) and solve for \( y(t) \)

• Zero Input
  • Set \( f(t) = 0 \) and \( y(t_0) = k \)
PHASOR CIRCUIT ANALYSIS

• Phasor
  • Complex number representing sinusoid with angular velocity $\omega$, eg $|F|e^{j\angle F}$
  • Example of Conversion: $f(t) = 2 \cos(2t + \frac{\pi}{3})$, $|F| = 2$, $\angle F = \frac{\pi}{3}$, Phasor=$2e^{j\frac{\pi}{3}}$

• Impedance (Z)
  • Ratio of Voltage to Current through circuit component, and extension of Ohm’s Law, $Z = \frac{V}{I}$
  • Impedance of Capacitor $Z_C = \frac{1}{j\omega C}$
  • Impedance of Inductor $Z_L = j\omega L$
  • Impedance of Resistor $Z = R$
**AVERAGE AND AVAILABLE POWER**

- **Average Power:**
  \[ P_{avg} = \frac{1}{2} Re\{VI^*\} = \frac{1}{2} Re\{V*I\} \]

- **Available Power:**
  \[ P_a = \frac{|V_s|^2}{8R_T}, \quad R_T = Re\{Z\} \]

- **Matches Load:**
  \[ Z_T = Z_L^* \]
FREQUENCY RESPONSE

- Relates input $f(t)$, output $y(t)$
- States how system reacts for frequency input $\omega$

- To Solve for $H(\omega)$:
  - Find $F(\omega)$ and $Y(\omega)$ from $f(t)$, $y(t)$
  - $H(\omega) = \frac{Y(\omega)}{F(\omega)}$

- Solve for $y(t)$ given $f(t)$ and $H(\omega)$
  - Find $F(\omega)$ from $f(t)$
  - $Y(\omega) = H(\omega)F(\omega)$
  - Find $y(t)$ from $Y(\omega)$
PERIODIC SIGNALS AND FOURIER SERIES

• Definition of Periodic Signal: $f(t - nT) = f(t)$ for some $T$ (period) and $n \in \mathbb{Z}$

• All frequencies represented by $kT$, $k \in \mathbb{Z}^+$

• Fundamental Frequency also called First Harmonic

• Fourier Series
  • All frequencies harmonically related
  • Way to decompose periodic signal into sinusoids
  • Absolutely Integrable: $\int |f(t)| dt < \infty$ (if satisfied then Fourier Coefficients $F_n$ are bounded)
  • Orthogonality: $\int_T e^{int} e^{imt} dt = 0$, $m \neq n$, crucial for Fourier Series
FOURIER SERIES (CONT.)

<table>
<thead>
<tr>
<th>$f(t)$, period $T = \frac{2\pi}{\omega_0}$</th>
<th>Form</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 nt}$</td>
<td>Exponential</td>
<td>$F_n = \frac{1}{T} \int_T f(t) e^{-j\omega_0 nt} dt$</td>
</tr>
<tr>
<td>$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos (n\omega_0 t) + b_n \sin (n\omega_0 t)$</td>
<td>Trigonometric</td>
<td>$a_n = F_n + F_{-n}$  ( b_n = j(F_n - F_{-n}) )</td>
</tr>
<tr>
<td>$\frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \cos (n\omega_0 t + \theta_n)$</td>
<td>Compact for real $f(t)$</td>
<td>$c_n = 2</td>
</tr>
</tbody>
</table>

Table 1: Fourier series forms.

<table>
<thead>
<tr>
<th>Name</th>
<th>Condition</th>
<th>Property:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Scaling</td>
<td>Constant $K$</td>
<td>$K f(t) \leftrightarrow K F_n$</td>
</tr>
<tr>
<td>2 Addition</td>
<td>$f(t) \leftrightarrow F_n$, $g(t) \leftrightarrow G_n$, ...</td>
<td>$f(t) + f(t) + \ldots \leftrightarrow F_n + G_n + \ldots$</td>
</tr>
<tr>
<td>3 Time shift</td>
<td>Delay $t_0$</td>
<td>$f(t - t_0) \leftrightarrow F_n e^{-j\omega_0 t_0}$</td>
</tr>
<tr>
<td>4 Derivative</td>
<td>Continuous $f(t)$</td>
<td>$\frac{df}{dt} \leftrightarrow j\omega_0 F_n$</td>
</tr>
<tr>
<td>5 Hermitean</td>
<td>Real $f(t)$</td>
<td>$F_n = F_{-n}^*$</td>
</tr>
<tr>
<td>6 Even function</td>
<td>$f(-t) = f(t)$</td>
<td>$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos (n\omega_0 t)$</td>
</tr>
<tr>
<td>7 Odd function</td>
<td>$f(-t) = -f(t)$</td>
<td>$f(t) = \sum_{n=1}^{\infty} b_n \sin (n\omega_0 t)$</td>
</tr>
<tr>
<td>8 Average power</td>
<td>$P \equiv \frac{1}{T} \int_T</td>
<td>f(t)</td>
</tr>
</tbody>
</table>

Table 2: Fourier series properties
QUESTIONS?
TIME FOR PAST PROBLEMS!
ACKNOWLEDGMENTS

A significant portion of these slides were based off of the HKN ECE 210 EXAM 2 Spring 2016 slides by Kaidong Peng, Julian Michaels, Seungjun Cho.