ECE 210 EXAM 2
HKN REVIEW SESSION

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STEADY STATE, TRANSIENT, ZERO STATE, ZERO INPUT

- Steady State
  - Does not go to 0 for $t \to \infty$, $\lim_{t\to\infty} f(t) = f_{ss}(t)$
  - Examples: Constants, Sinusoids
- Transient
  - Goes to zero over time, $\lim_{t\to\infty} f_{tr}(t) = 0$

Given ODE $\frac{dy}{dt} + y(t) = f(t)$

- Zero State
  - Set $y(t_0) = 0$ and solve for $y(t)$
- Zero Input
  - Set $f(t) = 0$ and $y(t_0) = k$

PHASOR CIRCUIT ANALYSIS

- Phasor
  - Complex number representing sinusoid with angular velocity ω, eg $|F|e^{j\angle F}$
  - Example of Conversion: $f(t) = 2\cos(2t + \frac{\pi}{3})$, $|F| = 2$, $\angle F = \frac{\pi}{3}$, Phasor=$2e^{j\frac{\pi}{3}}$
- Impedance (Z)
  - Ratio of Voltage to Current through circuit component, and extension of Ohm's Law, $Z = \frac{V}{I}$
  - Impedance of Capacitor $Z_C = \frac{1}{j\omega C}$
  - Impedance of Inductor $Z_L = j\omega L$
  - Impedance of Resistor $Z = R$

AVERAGE AND AVAILABLE POWER

- Average Power:

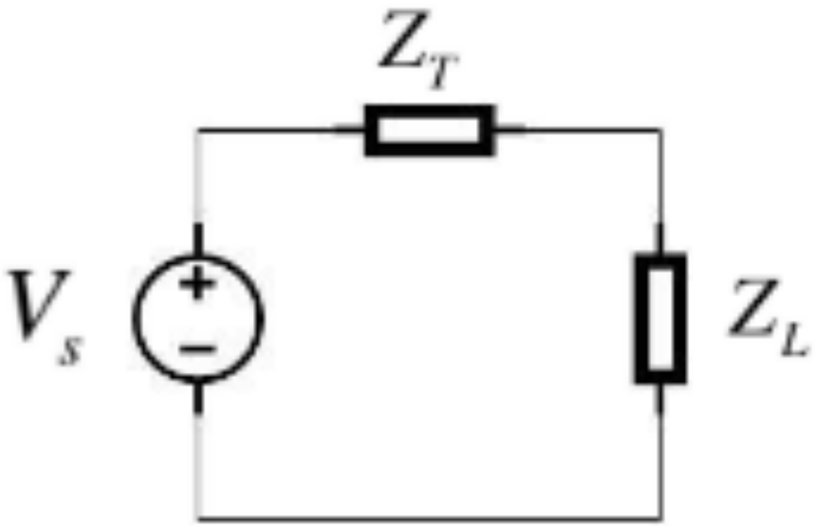
$$P_{avg} = \frac{1}{2}Re\{VI^*\} = \frac{1}{2}Re\{V^*I\}$$

- Available Power:

$$P_a = \frac{|V_s|^2}{8R_T}, R_T = Re\{Z\}$$

- Matches Load:

$$Z_T = {Z_L}^*$$



FREQUENCY RESPONSE

- Relates input f(t), output $y(t)$
- States how system reacts for frequency input ω
- To Solve for $H(\omega)$:
  - Find $\mathrm{F}(\omega)$ and $Y(\omega)$ from $f(t),\ y(t)$
  - $H(\omega) = \frac{Y(\omega)}{F(\omega)}$
- Solve for $y(t)$ given $f(t)$ and $H(\omega)$
  - Find $F(\omega)$ from $f(t)$
  - $Y(\omega) = H(\omega)F(\omega)$
  - Find $y(t)$ from $Y(\omega)$

PERIODIC SIGNALS AND FOURIER SERIES

- Definition of Periodic Signal: $f(t - nT) = f(t)$ for some T (period) and $n \in \mathbb{Z}$
- All frequencies represented by $kT, k \in \mathbb{Z}^+$
- Fundamental Frequency also called First Harmonic
- Fourier Series
  - All frequencies harmonically related
  - Way to decompose periodic signal into sinusoids
  - Absolutely Integrable: $\int |f(t)| dt < \infty$ (if satisfied then Fourier Coefficients F_n are bounded)
  - Orthongonality: $\int_T e^{jnt} e^{jmt}\, dt = 0, m \neq n$, crucial for Fourier Series

FOURIER SERIES (CONT.)

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\|F_n\|$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\,f(t) \leftrightarrow K\,F_n$
2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + f(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \|f(t)\|^2 dt = \sum_{n=-\infty}^{\infty} \|F_n\|^2$

Table 2: Fourier series properties

QUESTIONS?

TIME FOR PAST PROBLEMS!

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