ECE 210 Review Session

10/15/16

Overview

Part 1

- Steady/Transient State, Zero-State/Zero-Input
- Phasor Circuit Analysis
- Available power
- Frequency Response, Resonance
- Periodic Signals
- Fourier Series

Part 2

- Past Exam Problems

Steady State & Transient Responses

System response - sum of the steady state response and the transient response: $y(t) = y_{ss}(t) + y_{tr}(t)$

Steady State Response - $y_{ss}(t)$

- The component of the system response that is time invariant
- Constants: e.g. 5, j
- Sinusoids: e.g. $\cos(\omega t)$, $\sin(\omega t)$
- $y_{ss}(t) = \lim (t \rightarrow \text{inf})\ y(t)$

Transient Response - $y_{tr}(t)$

- The component of the system that decays to 0 as time proceeds
- Exponentials: e.g. ce^{-at}
- $y_{tr}(t) = y(t) - y_{ss}(t)$

Zero-State & Zero-Input Responses

System response - sum of the zero-state response and the zero-input response:
$y(t) = y_{zs}(t) + y_{zi}(t)$

Given the ODE describing the system: $\frac{\partial y(t)}{\partial t} + y(t) = f(t)$

Zero-State Response - $y_{zs}(t)$

- Solution to the ODE given the condition: $f(t) = f(t)$, $y(t_0) = 0$

Zero-Input Response - $y_{zi}(t)$

- Solution to the ODE given the condition: $f(t) = 0$, $y(t_0) = k_1$

Phasor Circuit Analysis

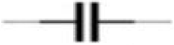
Phasor - Vector spinning at an arbitrary angular velocity (w). Usually written in polar form: $|F|e^{j\angle F}$

Conversion of a sinusoid to phasor form requires finding its magnitude and phase, ex: f(t) = 2cos(2t + pi/3); magnitude = 2, phase = pi/3 rad -> F = $2e^{j\angle pi/3}$

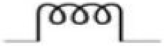
Impedance (Z) - Ratio of phasor Voltage to phasor Current passing through a circuit component. Z = V/I (extension of Ohm's Law to complex domain)

Impedance of electrical components

$Z = R$



$Z = \frac{1}{j\omega C}$



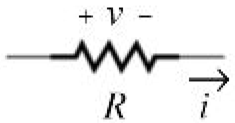
$Z = j\omega L$

From Chapter 2:

Maximum & Available Power

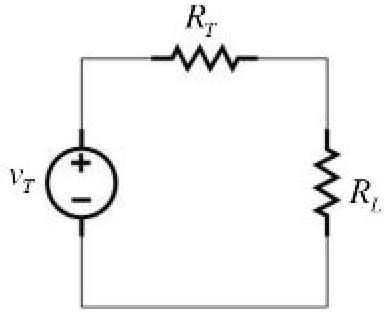
Absorbed Power

$$p = vi, \quad \sum_{all\ i} p_i = 0$$



Available Power

$$p_a = \frac{v_T^{\ 2}}{4R_T}$$



Matched Load

$$R_L = R_T$$

Average, Available Power

Average power:

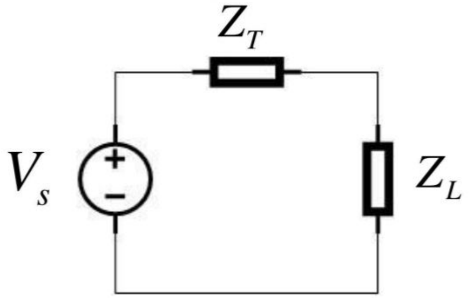
The average power delivered to a component is $P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\}$

Available Power:

The available power is $P_a = \frac{|V_s|^2}{8R_T}$, where $R_T = \mathrm{Re}\{Z_T\}$

Matches Load:

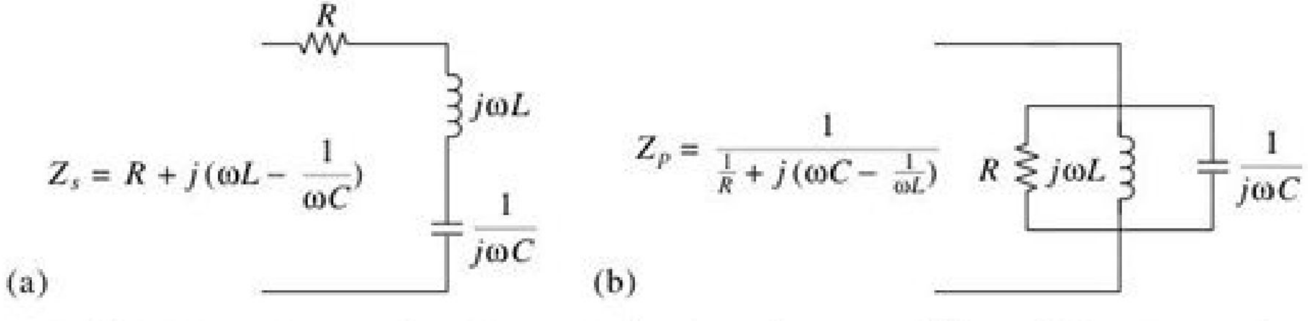
The available power is delivered to the load when $Z_L = Z_T{}^*$



Resonant Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

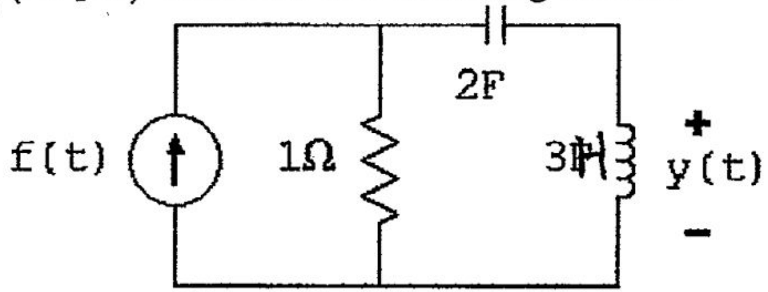
- Possible existence of steady-state co-sinusoidal oscillations in a source-free circuit
- In series resonance, equivalent impedance of L and C is effective short circuit
- In parallel resonance, equivalent impedance of L and C is effective open circuit



Frequency Response

- Given a system described by y(t) ~ f(t), take the fourier transform of both, and H(ω) = Y/F
  - f(t) -> |F|cos(ωt + <F)
  - y(t) -> |Y|cos(ωt + <Y)
- For ODEs - (d/dt) -> jω

3. (16 pts) Consider the following circuit.



(a) Determine the frequency response $H(\omega) = \frac{Y}{F}$.

Periodic Signals

- A signal is said to be periodic if there exists some delay t0 such that:

$$f(t-t0) = f(t)$$

- The period of the signal (T): the smallest non-zero value of t0.

Harmonically related frequencies

- All frequencies are positive integer multiples of the frequency of the original wave, known as the fundamental frequency (also called 1st harmonic).

- Note: Fourier series frequency components are harmonically related!

Fourier Series

A way to represent a periodic function as the sum of a set of simple sinusoid or complex exponential waves.

Fourier Series (Cont'd)

- Absolutely Integrable: $\int_T |f(t)|dt < \infty$

if the above inequality is satisfied, then the Fourier

Coefficients F_n must be bounded.

- Orthogonality

$\int (e^{jn\omega t})(e^{jm\omega t})^* dt = 0 \ (m \neq n)$

This condition is crucial to Fourier Series representation!

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$
$\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2\lvert F_n\rvert$ $\theta_n = \angle F_n$

Table 1: Fourier series forms.

	Name:	Condition:	Property:
1	Scaling	Constant K	$K\,f(t) \leftrightarrow K\,F_n$
2	Addition	$f(t) \leftrightarrow F_n,\ g(t) \leftrightarrow G_n,\ \ldots$	$f(t) + f(t) + \ldots \leftrightarrow F_n + G_n + \ldots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T}\int_T \lvert f(t)\rvert^2 dt = \sum_{n=-\infty}^{\infty} \lvert F_n\rvert^2$

Table 2: Fourier series properties

5. (24 pts) Consider the periodic signal $f(t) = \frac{3}{2} + 4cos^2(\frac{\pi}{4}t) + sin(\frac{3\pi}{4}t + \frac{\pi}{4})$ being the input to an LTI system having frequency response $H(\omega) = \frac{j\omega}{1+j\omega}$.

(a) Obtain the fundamental frequency ω_0 and period T of the input $f(t)$.

(Spring 2016 Exam 2 Problem 5)

$\omega_0 =$ ____________________

$T =$ ____________________

(b) For $n = 0, 1, -1, 2, -2$, obtain the exponential Fourier series coefficients of the input, F_n, and of the output Y_n.

Questions?

Acknowledgement

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